Mechanics of Fligt , Aircraft Time Responses:

1) Aircraft Free Motions

1-1) Longitudinal Free Motion:

$$\dot{X} = A.X$$

$$X(s) = (S.I - A)^{-1} X(0)$$

$$A = \begin{pmatrix} Xu & Xw & 0 & -g \\ Zu & Zw & U0 & 0 \\ Mbu & Mbw & Mbq & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad X(s) = \begin{pmatrix} u(s) \\ w(s) \\ q(s) \\ \theta(s) \end{pmatrix}$$

$$X = \frac{\begin{pmatrix} n11(s) & n12(s) & n13(s) & n14(s) \\ n21(s) & n22(s) & n23(s) & n24(s) \\ n31(s) & n32(s) & n33(s) & n34(s) \\ n41(s) & n42(s) & n43(s) & n44(s) \end{pmatrix}}{\left(s^4 + a1\cdot s^3 + a2\cdot s^2 + a3\cdot s + a4\right)} \cdot \begin{pmatrix} u(0) \\ w(0) \\ q(0) \\ \theta(0) \end{pmatrix}$$

$$n11(s) = s\cdot\left[s^2 - (Mq + Mwd\cdot U0 + Zw)\cdot s + (Zw\cdot Mq - Mw\cdot U0)\right]$$

$$n12(s) = Xw\cdot s^2 - Xw\cdot(Mq + Mwd\cdot U0)\cdot s - g\cdot(Mw + Mwd\cdot Zw)$$

$$n13(s) = s\cdot(U0\cdot Xw - g) + g\cdot Zw$$

$$n21(s) = Zu\cdot s^2 - (Zu\cdot Mq - Mu\cdot U0)\cdot s$$

$$n22(s) = s^3 - (Xu - Mq - Mwd\cdot U0)\cdot s^2 + Xu\cdot(Mq + Mwd\cdot U0)\cdot s + g\cdot(Mu + Mwd\cdot Zu)$$

$$n23(s) = U0\cdot s^2 - Xu\cdot U0\cdot s - g\cdot Zu$$

$$n24(s) = s\cdot[s\cdot(Mu + Mwd\cdot Zu) + (Zu\cdot Mwd - Zw\cdot Mu)]$$

$$n32(s) = s^2\cdot(Mw + Mwd\cdot Zw) - s\cdot[\, Xu\cdot Mw - Xw\cdot Mu + Mwd\cdot(Zw\cdot Xu - Zu\cdot Xw)\,]$$

$$n33(s) = s\cdot\left[s^2 - (Xu + Zw)\cdot s + (Xu\cdot Zw - Zu\cdot Xw)\right]$$

$$n41(s) = s\cdot(Mu + Mwd\cdot Zu) + (Zu\cdot Mw - Mu\cdot Zw)$$

$$n42(s) = s\cdot(Mw + Mwd\cdot Zu) + Zw\cdot Mu - Xu\cdot Mw + Mwd\cdot(Zu\cdot Xw - Xu\cdot Zw)$$

$$n43(s) = s^2 - (Xu + Zw)\cdot s + (Xu\cdot Zw - Zu\cdot Xw)$$

$$a1 = -(Xu + Mq + Zw + Mwd\cdot U0)$$

$$a2 = (Mq\cdot Zw - Mw\cdot U0 + Xu\cdot Zw - Zu\cdot Xw + Xu\cdot Mq + Xu\cdot U0\cdot Mwd)$$

$$a3 = -(Xu\cdot Zw\cdot Mq - Xu\cdot Mw\cdot U0 - Mq\cdot Zu\cdot Xw + Mu\cdot Xw\cdot U0 - g\cdot Mu - g\cdot Mwd\cdot Zu)$$

$$a4 = g\cdot(Zu\cdot Mw - Zw\cdot Mu)$$

Laplase of State Variables :

$$u(s) = \frac{n11(s)\cdot u0 + n12(s)\cdot w0 + n13(s)\cdot q0 + n14(s)\cdot \theta 0}{s^4 + a1\cdot s^3 + a2\cdot s^2 + a3\cdot s + a4}$$

$$w(s) = \frac{n21(s)\cdot u0 + n22(s)\cdot w0 + n23(s)\cdot q0 + n24(s)\cdot \theta 0}{s^4 + a1\cdot s^3 + a2\cdot s^2 + a3\cdot s + a4}$$

$$q(s) = \frac{n31(s)\cdot u0 + n32(s)\cdot w0 + n33(s)\cdot q0 + n34(s)\cdot \theta 0}{s^4 + a1\cdot s^3 + a2\cdot s^2 + a3\cdot s + a4}$$

$$\theta(s) = \frac{n41(s)\cdot u0 + n42(s)\cdot w0 + n43(s)\cdot q0 + n44(s)\cdot \theta 0}{s^4 + a1\cdot s^3 + a2\cdot s^2 + a3\cdot s + a4}$$

1-2) Lateral Motion :

$$\dot{X} = A.X$$

$$X(s) = (S.I - A)^{-1} X(0)$$

$$A = \begin{pmatrix} Yv & 0 & -1 & \frac{g}{U0} & 0 \\ L1\beta & L1p & L1r & 0 & 0 \\ N1\beta & N1p & N1r & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \qquad X(s) = \begin{pmatrix} \beta(s) \\ p(s) \\ r(s) \\ \phi(s) \\ \psi(s) \end{pmatrix}$$

$$X = \frac{\begin{pmatrix} n11(s) & n12(s) & n13(s) & n14(s) & n15(s) \\ n21(s) & n22(s) & n23(s) & n24(s) & n25(s) \\ n31(s) & n32(s) & n33(s) & n34(s) & n35(s) \\ n41(s) & n42(s) & n43(s) & n44(s) & n45(s) \\ n51(s) & n52(s) & n53(s) & n54(s) & n55(s) \end{pmatrix}}{\left[s\cdot\left(s^4 + d1\cdot s^3 + d2\cdot s^2 + d3\cdot s + d4\right)\right]} \cdot \begin{pmatrix} \beta(0) \\ p(0) \\ r(0) \\ \phi(0) \\ \psi(0) \end{pmatrix}$$

$$d1 = -(L1p + N1r + Yv)$$

$$d2 = \left(L1p\cdot N1r - L1r\cdot N1p + Yv\cdot L1p + Yv\cdot N1r + N1\beta\right)$$

$$d3 = \left(L1\beta\cdot N1p - L1p\cdot N1\beta - \frac{g}{U0}\cdot L1\beta - Yv\cdot L1p\cdot N1r + Yv\cdot L1r\cdot N1p\right)$$

$$d4 = \frac{g}{U0}\cdot\left(N1r\cdot L1\beta - L1r\cdot N1\beta\right)$$

$$n11(s) = s^2\cdot\left[s^2 - (L1p + N1r)\cdot s + (N1r\cdot L1p - L1r\cdot N1p)\right]$$

$$n12(s) = -s\cdot\left[\left(N1p - \frac{g}{U0}\right)\cdot s + \frac{N1r\cdot g}{U0}\right]$$

$$n13(s) = -s\cdot\left(s^2 - L1p - \frac{L1r}{U0}\right)$$

$$n21(s) = s^2\cdot\left[s\cdot L1\beta + \left(N1\beta\cdot L1r - L1\beta\cdot N1r\right)\right]$$

$$n22(s) = s^2\cdot\left[s^2 - (Yv + N1r)\cdot s + \left(Yv\cdot N1r + N1\beta\right)\right]$$

$$n23(s) = s^2\cdot\left[s\cdot L1r - \left(Yv\cdot L1r + L1\beta\right)\right]$$

$$n31(s) = s^2\cdot\left[s\cdot N1\beta + \left(L1\beta\cdot N1p - L1p\cdot N1\beta\right)\right]$$

$$n32(s) = s\cdot\left(s^2\cdot N1p - Yv\cdot N1p\cdot s + \frac{N1\beta\cdot g}{U0}\right)$$

$$n33(s) = s\cdot\left[s^3 - (Yv + L1p)\cdot s^2 + Yv\cdot L1p\cdot s - \frac{g}{U0}\cdot L1p\right]$$

$$n41(s) = s\cdot\left[s\cdot L1\beta + \left(L1\beta\cdot L1r - L1\beta\cdot N1r\right)\right]$$

$$n42(s) = s\cdot\left[s^2 - (Yv + N1r)\cdot s + \left(Yv\cdot N1r + N1\beta\right)\right]$$

$$n43(s) = s\cdot\left[s\cdot L1r - \left(Yv\cdot L1r + L1\beta\right)\right]$$

$$n51(s) = s^3\cdot\left[s\cdot N1\beta + \left(L1\beta\cdot N1p - N1\beta\cdot L1p\right)\right]$$

$$n52(s) = s^2\cdot\left(s^2\cdot N1p - Yv\cdot N1p\cdot s + g\cdot\frac{N1\beta}{U0}\right)$$

$$n53(s) = s^2\cdot\left[s^3 - (Yv + L1p)\cdot s^2 + Yv\cdot L1p\cdot s - \frac{g}{U0}\cdot L1\beta\right]$$

Laplase of State Variables :

$$\beta(s) = \frac{n11(s)\cdot\beta0 + n12(s)\cdot p0 + n13(s)\cdot r0 + n14(s)\cdot\phi0 + n15(s)\cdot\psi0}{s\cdot\left(s^4 + d1\cdot s^3 + d2\cdot s^2 + d3\cdot s + d4\right)}$$

$$p(s) = \frac{n21(s)\cdot\beta0 + n22(s)\cdot p0 + n23(s)\cdot r0 + n24(s)\cdot\phi0 + n25(s)\cdot\psi0}{s\cdot\left(s^4 + d1\cdot s^3 + d2\cdot s^2 + d3\cdot s + d4\right)}$$

$$r(s) = \frac{n31(s)\cdot\beta0 + n32(s)\cdot p0 + n33(s)\cdot r0 + n34(s)\cdot\phi0 + n35(s)\cdot\psi0}{s\cdot\left(s^4 + d1\cdot s^3 + d2\cdot s^2 + d3\cdot s + d4\right)}$$

$$\phi(s) = \frac{n41(s)\cdot\beta0 + n42(s)\cdot p0 + n43(s)\cdot r0 + n44(s)\cdot\phi0 + n45(s)\cdot\psi0}{s\cdot\left(s^4 + d1\cdot s^3 + d2\cdot s^2 + d3\cdot s + d4\right)}$$

$$\psi(s) = \frac{n51(s)\cdot\beta0 + n52(s)\cdot p0 + n53(s)\cdot r0 + n54(s)\cdot\phi0 + n55(s)\cdot\psi0}{s\cdot\left(s^4 + d1\cdot s^3 + d2\cdot s^2 + d3\cdot s + d4\right)}$$

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مشتق پایداری	ضریب پایداری (پارامتر بی بعد)
X_i	C_{Xi}

L = $\frac{1}{2}\cdot\rho\cdot U0^2\cdot S\cdot CL$ D = $\frac{1}{2}\cdot\rho\cdot U0^2\cdot S\cdot CD$ M = $\frac{1}{2}\cdot\rho\cdot U0^2\cdot S\cdot Cm$

تبدیل مشتقات پایداری و ضرایب پایداری یک هواپیما	
حرکت طولی	حرکت عرضی

$$X_u = -\frac{\rho U_0 S}{m}(C_{Du} + 2C_{D0})$$

$$X_w = \frac{\rho U_0 S}{2m}(-C_{D\alpha} + C_{L0})$$

$$Z_u = -\frac{\rho U_0 S}{m}(C_{Lu} + 2C_{L0})$$

$$Z_w = -\frac{\rho U_0 S}{2m}(C_{L\alpha} + C_{D0})$$

$$M_u = \frac{\rho U_0 S\bar{c}}{I_{yy}}(C_{mu} + C_{m0})$$

$$M_w = \frac{\rho S U_0 \bar{c}}{2I_{yy}} C_{m\alpha}$$

$$M_{\dot{w}} = \frac{\rho S \bar{c}^2}{4I_{yy}} C_{m\dot{\alpha}}$$

$$M_q = \frac{\rho U_0 \bar{c}^2}{4I_{yy}} C_{mq}$$

$$Z_{\delta E} = -\frac{\rho U_0{}^2 S}{2m} C_{L\delta E}$$

$$M_{\delta E} = \frac{\rho U_0{}^2 S\bar{c}}{2m} C_{m\delta E}$$

$$Y_\beta = Y_v = \frac{\rho U_0 S}{m} C_{y\beta}$$

$$L_\beta = U_0 L_v = \frac{\rho U_0{}^2 Sb}{2I_{xx}} C_{l\beta}$$

$$N_\beta = \frac{\rho U_0{}^2 Sb}{2I_{zz}} C_{n\beta}$$

$$L_p = \frac{\rho U_0 Sb^2}{4I_{xx}} C_{lp}$$

$$N_p = \frac{\rho U_0 Sb^2}{4I_{zz}} C_{np}$$

$$L_r = \frac{\rho U_0 Sb^2}{4I_{xx}} C_{lr}$$

$$N_r = \frac{\rho U_0 Sb^2}{4I_{zz}} C_{np}$$

$$Y_\delta = \frac{\rho U_0{}^2 S}{2m} C_{y\delta}$$

$$L_\delta = \frac{\rho U_0{}^2 Sb}{2I_{xx}} C_{l\delta}$$

$$N_\delta = \frac{\rho U_0{}^2 Sb}{2I_{zz}} C_{n\delta}$$

2) Aircraft Excited Motions

2-1) Longitudinal Transfer Functions :

$$\dot{X} = A.X + B.u$$
$$X(s) = (S.I - A)^{-1} B.u$$

$$A = \begin{pmatrix} Xu & Xw & 0 & -g \\ Zu & Zw & U0 & 0 \\ Mbu & Mbw & Mbq & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad B = \begin{pmatrix} X\delta e & X\delta th \\ Z\delta e & Z\delta th \\ Mb\delta e & M\delta th \\ 0 & 0 \end{pmatrix} \quad u = \begin{pmatrix} \delta e \\ \delta th \end{pmatrix}$$

Transfer Functions of Langitudinal State Variables :

$$\frac{u(s)}{\delta e(s)} = \frac{n11(s)\cdot X\delta e + n12(s)\cdot Z\delta e + n13(s)\cdot Mb\delta e}{s^4 + a1\cdot s^3 + a2\cdot s^2 + a3\cdot s + a4}$$

$$\frac{w(s)}{\delta e(s)} = \frac{n21(s)\cdot X\delta e + n22(s)\cdot Z\delta e + n23(s)\cdot Mb\delta e}{s^4 + a1\cdot s^3 + a2\cdot s^2 + a3\cdot s + a4}$$

$$\frac{q(s)}{\delta e(s)} = \frac{n31(s)\cdot X\delta e + n32(s)\cdot Z\delta e + n33(s)\cdot Mb\delta e}{s^4 + a1\cdot s^3 + a2\cdot s^2 + a3\cdot s + a4}$$

$$\frac{\theta(s)}{\delta e(s)} = \frac{n41(s)\cdot X\delta e + n42(s)\cdot Z\delta e + n43(s)\cdot Mb\delta e}{s^4 + a1\cdot s^3 + a2\cdot s^2 + a3\cdot s + a4}$$

$$\frac{u(s)}{\delta th(s)} = \frac{n11(s)\cdot X\delta th}{s^4 + a1\cdot s^3 + a2\cdot s^2 + a3\cdot s + a4}$$

$$\frac{w(s)}{\delta th(s)} = \frac{n21(s)\cdot X\delta th}{s^4 + a1\cdot s^3 + a2\cdot s^2 + a3\cdot s + a4}$$

$$\frac{q(s)}{\delta th(s)} = \frac{n31(s)\cdot X\delta th}{s^4 + a1\cdot s^3 + a2\cdot s^2 + a3\cdot s + a4}$$

$$\frac{\theta(s)}{\delta th(s)} = \frac{n41(s)\cdot X\delta th}{s^4 + a1\cdot s^3 + a2\cdot s^2 + a3\cdot s + a4}$$

$$u(s) = \frac{u(s)}{\delta e(s)}\cdot\delta e(s) + \frac{u(s)}{\delta th(s)}\cdot\delta th(s)$$

2-2) Lateral Transfer Functions :

$$\dot{X} = A.X + B.u$$
$$X(s) = (S.I - A)^{-1} B.u$$

$$A = \begin{pmatrix} Yv & 0 & -1 & \frac{g}{U0} & 0 \\ L1\beta & L1p & L1r & 0 & 0 \\ N1\beta & N1p & N1r & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 & Y1\delta r \\ L1\delta a & L1\delta r \\ N1\delta a & N1\delta r \\ 0 & 0 \end{pmatrix} \quad u = \begin{pmatrix} \delta a \\ \delta r \end{pmatrix}$$

$$\frac{\beta(s)}{\delta a(s)} = \frac{n12(s)\cdot L1\delta a + n13(s)\cdot N1\delta a}{s\cdot\left(s^4 + d1\cdot s^3 + d2\cdot s^2 + d3\cdot s + d4\right)}$$

$$\frac{p(s)}{\delta a(s)} = \frac{n22(s)\cdot L1\delta a + n23(s)\cdot N1\delta a}{s\cdot\left(s^4 + d1\cdot s^3 + d2\cdot s^2 + d3\cdot s + d4\right)}$$

$$\frac{r(s)}{\delta a(s)} = \frac{n32(s)\cdot L1\delta a + n33(s)\cdot N1\delta a}{s\cdot\left(s^4 + d1\cdot s^3 + d2\cdot s^2 + d3\cdot s + d4\right)}$$

$$\frac{\phi(s)}{\delta a(s)} = \frac{n42(s)\cdot L1\delta a + n43(s)\cdot N1\delta a}{s\cdot\left(s^4 + d1\cdot s^3 + d2\cdot s^2 + d3\cdot s + d4\right)}$$

$$\frac{\psi(s)}{\delta a(s)} = \frac{n52(s)\cdot L1\delta a + n53(s)\cdot N1\delta a}{s\cdot\left(s^4 + d1\cdot s^3 + d2\cdot s^2 + d3\cdot s + d4\right)}$$

$$\frac{\beta(s)}{\delta R(s)} = \frac{n11(s)\cdot Y1\delta R + n12(s)\cdot L1\delta R + n13(s)\cdot N1\delta R}{s\cdot\left(s^4 + d1\cdot s^3 + d2\cdot s^2 + d3\cdot s + d4\right)}$$

$$\frac{p(s)}{\delta R(s)} = \frac{n21(s)\cdot Y1\delta R + n22(s)\cdot L1\delta R + n23(s)\cdot N1\delta R}{s\cdot\left(s^4 + d1\cdot s^3 + d2\cdot s^2 + d3\cdot s + d4\right)}$$

$$\frac{r(s)}{\delta R(s)} = \frac{n31(s)\cdot Y1\delta R + n32(s)\cdot L1\delta R + n33(s)\cdot N1\delta R}{s\cdot\left(s^4 + d1\cdot s^3 + d2\cdot s^2 + d3\cdot s + d4\right)}$$

$$\frac{\phi(s)}{\delta R(s)} = \frac{n41(s)\cdot Y1\delta R + n42(s)\cdot L1\delta R + n43(s)\cdot N1\delta R}{s\cdot\left(s^4 + d1\cdot s^3 + d2\cdot s^2 + d3\cdot s + d4\right)}$$

$$\frac{\psi(s)}{\delta R(s)} = \frac{n51(s)\cdot Y1\delta R + n52(s)\cdot L1\delta R + n53(s)\cdot N1\delta R}{s\cdot\left(s^4 + d1\cdot s^3 + d2\cdot s^2 + d3\cdot s + d4\right)}$$

$$\beta(s) = \frac{\beta(s)}{\delta a(s)}\cdot\delta a(s) + \frac{\beta(s)}{\delta R(s)}\cdot\delta R(s)$$